



The Multiplier

A simple Keynesian model of the economy with no Government or International Trade can be represented as:

$$Y = C + I \quad (1)$$

where Y is equilibrium output (income), C is aggregate consumption, and I is aggregate investment. Aggregate consumption, or total expenditure by households on final goods and services, is determined by autonomous consumption (a), or the rate of consumer expenditure independent of disposable income, and the marginal propensity to consume ($b = mpc$), which is the part of each additional dollar of disposable income that is spent on consumption. Thus, the consumption function is:

$$C = a + bY \quad (2)$$

Investment is determined by a complex of factors such as expectations of investors and lending institutions, business confidence, political climate, and so on (see other handout). For present purposes, what is important is that investment is autonomous—that is, independent—of income.

Substituting equation (2) into equation (1), we get:

$$Y = a + bY + I \quad (3)$$

Subtracting bY from both sides:

$$Y - bY = a + I \quad (4)$$

Factoring out the Y from the left hand side of equation (4)

$$Y(1 - b) = a + I \quad (5)$$

Dividing both sides by $(1 - b)$:

$$Y = \frac{1}{1-b} (a + I) \quad (6)$$

where $1/(1-b)$, is the multiplier, or the feedback mechanism that amplifies any initial increase (injection) or decrease (withdrawal) in aggregate demand. Therefore, the equilibrium level of output (income) is determined by the multiplier and total injections. The total injections in this simple model are $a + I$.

If we fill in the values from our example, then:

$$C = 100 + .75Y$$

$$I = 300$$

$$Y = 100 + .75Y + 300$$

$$Y - .75Y = 100 + 300$$

$$Y (1 - .75) = 100 + 300$$

$$Y (.25) = 100 + 300$$

$$Y = (1/.25) \times 400$$

$$Y = 4 \times 400$$

$$Y_e = \$1600 \text{ billion}$$

Isn't that special?