

Keynesian Supermultiplier

National income accounting equation:

$$Y = C + I + G + X - M \quad (1)$$

Consumption function includes autonomous consumption, a , plus the marginal propensity to consume, b , times disposable income:

$$C = a + bY_d \quad (2)$$

Disposable income is aggregate national income less the tax bill, T :

$$Y_d = Y - T \quad (3)$$

Assuming a flat rate income tax, t , the tax bill is equal to the tax rate times aggregate national income:

$$T = tY \quad (4)$$

Substituting equation 4 into equation 3 gives:

$$Y_d = Y - tY \quad (5)$$

Investment may be thought of as having an autonomous component, d , but also a component that is a function of income (not disposable income). So we can introduce the marginal propensity to invest, e :

$$I = d + eY \quad (6)$$

Government spending remains autonomous:

$$G = G \quad (7)$$

Aggregate exports are also exogenous, since it is determined by other nations' demand:

$$X = X \quad (8)$$

Imports, however, may be thought of as having an autonomous component, f , and a component that depends on disposable income (marginal propensity to import, m):

$$M = f + mY_d \quad (9)$$

Substituting equations 2, 6, and 9 into equation 1 gives:

$$Y = a + bY_d + d + eY + G + X - (f + mY_d) \quad (10)$$

Substituting equation 5 into equation 10 gives:

$$Y = a + b(Y - tY) + d + eY + G + X - f - m(Y - tY) \quad (11)$$

Multiplying through (watch those signs!):

$$Y = a + bY - btY + d + eY + G + X - f - mY + mtY \quad (12)$$

Solving for Y, take all the Ys over to the other side (watch those signs!):

$$Y - bY + btY - eY + mY - mtY = a + d + G + X - f \quad (13)$$

Factor out the Y:

$$Y(1 - b + bt - e + m - mt) = a + d + G + X - f \quad (14)$$

Isolate the Y:

$$Y = 1/(1 - b + bt - e + m - mt) (a + d + G + X - f) \quad (15)$$

This tells us that equilibrium aggregate output (income) is equal to the supermultiplier:

$$1/(1 - b + bt - e + m - mt) \quad (16)$$

times autonomous expenditures:

$$(a + d + G + X - f) \quad (17)$$

We can see from this the impact of certain variables on output and income. For example, the larger autonomous imports, the smaller will be national output and income. Likewise, the larger the marginal propensity to import, the smaller will be the multiplier. On the other hand, the larger the propensity to invest, the larger will be the multiplier.

Once we solve for Y, we can calculate $T = tY$, and with T calculate $Y_d = Y - T$. With Y_d we can calculate $C = a + bY_d$ and $S = -a + (1-b)Y_d$ (double check S with $Y_d - C$), as well as $M = f + mY_d$. $I = d + eY$, and G and X are given.

Remember that once we have government and foreign trade, $S = I$ is no longer the macroeconomic equilibrium condition. Rather, total injections into the expenditure flow, $I + G + X$, equal total withdrawals (or leakages) from the expenditure flow, $S + T + M$:

$$I + G + X = S + T + M \quad (18)$$

We can check the relation of G and T to see if the government is running a budget deficit, surplus, or a balanced budget. We can check the relation of X and M to see if there is a trade deficit, trade surplus, or balanced trade. The relation of S and I will show whether the private sector is in surplus or deficit. Of course, the government budget will be equal to the non-government balance. If, as currently in the U.S., the government is running a surplus and there is a trade deficit (foreign sector surplus), the private sector must be in deficit:

$$(T - G) + (M - X) = (I - S) \quad (19)$$

{Public Sector Surplus} + {Foreign Sector Surplus} = {Private Sector Deficit}