THE TYRANNY OF THE IDENTITY: GROWTH ACCOUNTING REVISITED

Jesus Felipe
Economics and Research Department
Asian Development Bank
Manila (Philippines)
e-mail: jfelipe@adb.org

JSL McCombie
Centre for Economic and Public Policy
University of Cambridge
Cambridge (UK)
e-mail: jslm2@hermes.cam.ac.uk

September 8, 2005

Abstract: This paper shows, first, that the recent proposal by Barro (1999) and Hsieh (1999, 2002) to perform growth accounting by directly differentiating the NIPA identity is simply an exercise in the manipulation of an accounting identity without any foundation. Second, simulations show that growth accounting performed with aggregate data is not equivalent to the true rate of technological progress implied by the micro data. Third, we conclude that the rate of total factor productivity growth usually estimated is simply a measure of distributional income.

JEL Classification: O11, O16, O47, O53

We are grateful to Rana Hasan, Franklin M. Fisher and the participants in the Conference on "Understanding Economic Growth: New Directions in Theory and Policy", Cambridge, U.K., September 1-3, 2005, for their comments and suggestions. The usual disclaimer applies. This paper represents the views of the authors and does not represent those of the Asian Development Bank, its Executive Directors, or the countries that they represent.

THE TYRANNY OF THE IDENTITY: GROWTH ACCOUNTING REVISITED

"The conclusion to be drawn from this brief discussion is that aggregation is a serious problem affecting the magnitude, the stability, and the dynamic changes of total factor productivity. We need to be cautious in interpreting the results that depend on the existence and specification of the aggregate production function…That the use of the aggregate production function gives reasonably good estimates of factor productivity is due mainly to the narrow range of movement of aggregate data rather than the solid foundation of the function. In fact, the aggregate production function does not have a conceptual reality of its own."

Nadiri (1970, p.1145-1146).

1. INTRODUCTION

A series of papers (Barro 1999; Hsieh 1999, 2002) have recently argued that growth accounting exercises, specifically the derivation of the dual measure of total factor productivity growth (TFPG), can be performed by simply differentiating the National Income and Product Accounts (NIPA) identity. According to this, value added equals the wage bill plus total profits, that is,

$$V_t \equiv Y_t - M_t \equiv W_t + \Pi_t \equiv w_t L_t + r_t J_t \qquad (1)$$

where V is value added, Y is gross output, M is the value of intermediate materials, W is the total wage bill, Π denotes total profits (operating surplus in the NIPA terminology), w is the average real wage rate, L is employment, r is the average ex-post profit rate (this is discussed below), and J is the deflated value of the stock of capital, with all variables expressed in real terms. Expressing equation (1) in growth rates yields:

$$\hat{V}_t \equiv a_t \hat{w}_t + (1 - a_t)\hat{r}_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t$$

$$\equiv TFPG_t + a_t \hat{L}_t + (1 - a_t) \hat{J}_t \qquad (2)$$

where ^ denotes a proportional growth rate, $a_t \equiv w_t L_t / V_t$ is the share of labor in output, $1 - a_t \equiv r_t J_t / V_t$ is the share of capital. Equation (2) can be rearranged to give:

$$TFPG_t \equiv \hat{V}_t - (1 - a_t) \hat{J}_t - a_t \hat{L}_t$$

$$\equiv a_t \hat{w}_t + (1 - a_t) \hat{r}_t \qquad (3)$$

which is interpreted as the rate of total factor productivity growth.

This approach to growth accounting stands in marked contrast to the arguments of Felipe and McCombie (2003). There it was argued that precisely *because* of the existence of the accounting identity, growth accounting exercises amount to no more than manipulations of the ex-post national income accounting identity, and, as such, they are tautologies without necessarily any behavioral content. This accounting identity, as shown above, is the one that relates the value of output to the sum of the wage bill plus profits. The reason for the problem that Felipe and McCombie (2003) highlight is that there is an important difference between using physical data, only available at a disaggregated level, and using data expressed in value terms, the type data used at any level of aggregation. For purposes of growth accounting exercises and for the computation of the rate of TFPG in particular, data in value terms on output, capital and labor (a physical quantity) are related through the above accounting identity. As a consequence, this paper shows that the rate of TFPG estimated with aggregate data has little relation with the rate of technical progress estimated using physical data. It should be noted, however, that estimates of TFPG have recently been calculated using physical output at the seven-digit SIC product classification (Foster *et al.* 2005). We briefly discuss this work and argue that it does not avoid the problems that this paper discusses.

This paper has a dual purpose. The first one is to provide an evaluation of what superficially seems to be a useful methodological; that is, to derive the rate of TFPG from the NIPA, as suggested by Barro and Hsieh. The second purpose is to study using simulations whether the rate of total factor productivity growth (TFPG) using aggregate data (in value terms) is equivalent to the true rate of technical progress (φ) estimated using disaggregated data (in physical terms). The difference between data in physical terms and in value terms is prices, typically unobserved.

The two objectives of the paper are, in fact, related. Indeed, as we shall see, the rate of TFPG is determined by the accounting identity. In doing this, we raise some fundamental methodological problems of the growth accounting approach, and extend the arguments in Felipe and McCombie (2003) in the context of the well-known and seemingly inexhaustible debate of the sources of growth in East Asia that began with the growth accounting studies of Young (1992, 1995) and econometric analysis of Kim and Lau (1994), and which Hsieh (1999, 2002) has revisited.[1] Nevertheless, the argument goes beyond this particular debate and for this reason we also take Barro's (1999) survey on growth accounting, who also adopts the same approach as Hsieh, as our starting point. Fernald and Neiman (2003) also discuss the debate about the sources of growth in East Asia and carry out their growth accounting analysis from the NIPA accounting identity. Hart (1996) also shares the same view. We discuss this below.

Hsieh's (1999, 2002) approach is based on the assumption that growth accounting can be performed by simply differentiating the NIPA identity, as shown above.[2] The use of the dual is not new and, indeed, forms the rationale behind Jorgenson and Griliches (1967) seminal growth accounting study. As Hsieh puts it:

---

1 Hsieh's (1999, 2002) central tenet about the East Asian economies is the observation that if the capital-output ratio in the NIEs indeed increased as much as the data indicates, the return to capital should have fallen dramatically as capital accumulation encounters diminishing returns. This is a point also made by Nelson and Pack (1999) and elaborated upon by Felipe and McCombie (2001). Hsieh's calculations and data sources have been subject to criticism by Young (1998).

2 Hsieh (2002, p.502) argues: "It is useful to think about this as an accounting identity."

> "with only the *condition* that output equals factor incomes, we have the result that the primal and dual measures of the Solow residual are equal. No other assumptions are needed for this result: we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products. We do not even need to assume that the data is correct. For example, if the capital stock data is wrong, the primal estimate of the Solow residual will clearly be a biased estimate of aggregate technological change. However, as long as the output and factor price data are consistently wrong, the dual measure of the Solow residual will be exactly equal to the primal measure, and consequently, equally biased.
>
> The two measures of the Solow residual *can differ* when national output exceeds the payments to capital and labor" (Hsieh 2002, p.505; italics added).

Barro (1999) also concurs: "the dual approach can be derived readily from the equality between output and factor income" (Barro 1999, p.123). To show this, he writes the income accounting identity, differentiates it, and expresses it in terms of growth rates (Barro 1999, equations (7) and (8)). Barro and Hsieh agree that: "it is important to recognize that the derivation of equation (8) [the growth accounting equation in his paper] uses only the condition $V_t = r_t J_t + w_t L_t$. No assumptions were made about the relations of factor prices to social marginal products or about the form of the production function" (Barro 1999, pp.123).[3] Barro continues: "If the condition $V_t = r_t J_t + w_t L_t$ holds, then the primal and dual estimates of TFP growth inevitably coincide […] If the condition $V_t = w_t L_t + r_t J_t$ holds, then the discrepancy between the primal and dual estimates of TFP has to reflect the use of different data in the two calculations" (Barro 1999, p.123-124).

In the light of Barro's (1999) and Hsieh's (1999, 2002) papers, it is worth elaborating their arguments and contrasting them with those of Felipe and McCombie (2003). While Hsieh and Barro acknowledge that they manipulate an identity, not withstanding the quotation above it is clear that there are neoclassical assumptions

[3] The notation in Barro's quotations has been changed to make it consistent with that in this paper.

implicit in their analysis.[4] Our argument is that because of the underlying accounting identity in value terms, it is not possible to test these assumptions (and potentially refute them) and hence to dichotomize the growth of output into that caused by the (weighted) growth of the factors of production and a residual, which is interpreted as the rate of technical change or, more generally, as the rate of increase in efficiency of the economy. We conclude that the procedure Hsieh and Barro propose is without any foundation.

The rest of the paper is structured as follows. Section 2 summarizes the Barro-Hsieh approach to growth accounting. The section also reminds the reader of the insurmountable problems posed by the Cambridge Capital Theory debates and the aggregation problem for the notion of aggregate production function. Section 3 explains why most often estimation of aggregate production functions with time series leads to poor results; and shows why estimating econometrically aggregate productions, if done correctly, must always lead to the same paradoxical finding. Section 4 uses a series of simple simulations to shed light on what growth accounting is actually measuring, and considers whether or not the rate of TFPG using aggregate data is equivalent to the true rate of technical progress estimated using disaggregated data and then added up. The answer is no. Given these results, we propose in section 5 a reinterpretation of growth accounting exercises and of the rate of TFPG consistent with the non-existence of the aggregate production function. We also briefly consider Hart's (1996) and Foster *et al.'s* (2005) analyses. Section 6 concludes.

2. THE NIPA, GROWTH ACCOUNTING AND THE BARRO-HSIEH APPROACH

The Barro-Hsieh thesis is that growth accounting exercises can be directly performed using data from the NIPA, according to which value added equals the payments to the factors of production, as expressed in equation (1). From our point of view, it is important to emphasize that the symbol "≡" indicates that expression (1) is an

[4] This is very clear in Hsieh (2002, p.502), where he refers to the "rental price of capital" and the "marginal product of capital." These concepts are theory dependent.

accounting identity, which has, *per se,* no behavioral implications and will hold irrespective of the state of competition and whether or not an aggregate production function actually exists. It must be added that the NIPA does not provide the decomposition on the right-hand side of (1), but only the aggregate sum of the payments to the factors of production, that is, $V_t \equiv W_t + \Pi_t$. The decomposition of the wage bill (W) and the operating surplus (Π) into the products of the factor prices times the "quantities" is definitional, but artificial. Moreover, it will always be true that the wage bill can be written as the product of the average wage (*w*) rate times employment (*L*). On the other hand, whether the wage rate equals the marginal product of labor or not, is quite another matter. Nevertheless, data on labor and the constant price value of the capital stock are available from the statistical offices such as the US Bureau of Labor Statistics and the OECD.

Why do Hsieh and Barro argue that growth accounting can be performed by simply thinking in terms of the NIPA? Equation (3), *under the usual neoclassical assumptions*, is formally equivalent to the residual measure of total TFPG. The first part, $TFPG_t = \hat{V}_t - (1 - a_t)\hat{J}_t - a_t \hat{L}_t$, is identical to the growth accounting equation derived from a neoclassical aggregate production function, imposing the conditions for producer equilibrium, and is referred to as the *primal* measure of TFPG. The second part of equation (3), $TFPG_t \equiv a_t \hat{w}_t + (1 - a_t)\hat{r}_t$, is the *dual* measure of TFPG, derived in neoclassical economics from the cost function. The latter is the expression Hsieh used in his empirical analyses.

What are the neoclassical assumptions referred to above that lead Barro and Hsieh to argue that equation (1) allows a growth accounting exercise? Implicit in the neoclassical approach is the existence of an aggregate production function with Hicks neutral technical change, i.e., $V = A(t)F(L,J)$. Factors are paid their marginal products (f_i, *i*=*L, J*) and Euler's theorem gives the result that $V = f_L L + f_J J$ where f_L and f_J are equal to the factor prices, which equal the marginal social products. Moreover, the output elasticities are equal to the relevant factor shares. In our view, the most controversial assumption underlying the growth accounting approach is the existence of such an aggregate production function. This is a question not even considered in orthodox analyses. We return to this issue at the end of the section.

If the aggregate production function *does exist*, it may be written generally in growth rate form as

$$\hat{V}_t = TFPG_t + \alpha_t \hat{L}_t + \beta_t \hat{J}_t \qquad (4)$$

where α and β are the output elasticities of labor and capital, respectively; and $TFPG_t$ is interpreted as the growth rate of technical progress, or the *Solow residual*. As there are usually no reliable estimates of α or β, by invoking the first order conditions (from profit maximization and competitive markets), the factor elasticities are taken to equal the appropriate factor shares, i.e., $\alpha_t = a_t$ and $\beta_t = 1 - a_t$. Hence:

$$\hat{V}_t = TFPG_t + a_t \hat{L}_t + (1 - a_t)\hat{J}_t \qquad (5)$$

These are not innocuous assumptions. The interpretation of $TFPG_t$ in equations (2) and (3) as the rate of technological progress follows directly from a comparison with (5); otherwise, on what grounds can $TFPG_t$ in equations (2) and (3) be referred to as the rate of technical progress? As we have noted above, the aggregate production function, together with the conditions for producer equilibrium (the first-order conditions), provide the underlying theory for the standard interpretation of the growth accounting exercise. In fact, it is argued that the aggregate production function provides a theory of the income side of the NIPA (Prescott 1998, p.532). This follows from the alleged link that is believed to exist between the accounting identity, the aggregate production function (assumed to be homogeneous of first degree, and to have positive first order partial derivatives and negative second order partial derivatives) and, as mentioned above, Euler's theorem (Hulten 2000, p.11). The theory implies that $V = f_L L + f_J J$. From the first-order conditions $f_L = (\partial F / \partial L) = w$ and

$f_J = (\partial F / \partial J) = \nu$, it follows that $V = wL + \nu J$, (where ν is the rental price of capital) which is thought to be the identity (1). Somehow, the neoclassical framework considers that the production function, through Euler's theorem, *implies* the identity. But the important thing to note is that under this argument, this identity has become a "virtual" identity, one that holds *if and only if* the theory under which it was built holds.

On the other hand, our argument is that the wage bill and total profits can be split into the respective products (i.e., equation (1)) without recourse to any theory and do not require that wage and profit rates equal their respective marginal products. Our argument (Felipe and McCombie 2003) is that the identity $V \equiv W + \Pi \equiv wL + rJ$ holds irrespective of whether any of these conditions are fulfilled. Hsieh's quotation cited above that "we do not need any assumption about the form of the production function, bias of technological change, or relationship between factor prices and their social marginal products" is misleading to the extent that it could be interpreted as implying that none of these conditions are required for growth accounting. What it simply means is that if we have an identity $X \equiv Y + Z$, then this may be expressed as $\theta\hat{Y} \equiv \hat{X} - (1-\theta)\hat{Z}$, where $\theta \equiv Y/X$. If we term the right-hand side the "primal", then by definition it must equal the left-hand side, whether it is called the "dual" or anything else. Consequently, any measurement error on either the right-hand side or the left-hand side of the equation in growth rate form must lead to an equal error on the opposite side. But this should not lead to overlook the necessary assumptions that underlie the growth accounting approach, if either the dual or the primal is to be interpreted as a measure of technical progress, or of the increase in efficiency.

If there are increasing returns to scale, then the Solow residual could still be calculated as the primal as $TFPG = \hat{V} - \alpha\hat{L} - \beta\hat{J}$ ($\alpha + \beta > 1$), but clearly this residual would not be equal to the weighted average of the growth rates of the wage and profit rates (i.e., $a_t\hat{w}_t + (1-a_t)\hat{r}_t$). Factors of production do need to be paid their marginal social values if the Solow residual is to be given an economic interpretation. Moreover, while it is true that we do not need to know the precise functional form of the aggregate production function, we do need to assume that it exists.

Consequently, bearing this qualification in mind, from the accounting identity, if a consistent data set (i.e., one that makes the identity (1) hold) is used for calculating the dual and the primal measures of TFPG, then, by definition, they must be equal. The identity consists of five variables, namely, *V*, *w*, *L*, *r*, and *J*. If values of each are obtained "independently" then it is possible that the identity, equation (1), will not hold because of measurement error. For this reason, to ensure consistency, one of the variables must be obtained residually.

The NIPA provides data on *V*, *W* and *Π*; data on *w* and *L* can be obtained from wage and labor force statistics; and an estimate of *J* may be obtained by the perpetual inventory method (some statistical offices also compute it). Consequently, it is often *r* that is obtained residually. But if there were independent estimates of *r* published or calculated, one could equally obtain *J* (or any of the other variables for that matter) residually.[5] However, the two approaches could differ if the identity is not consistent. For example, if *r* and *J* are calculated independently, it may well be that $r_t J_t \neq V_t - w_t L_t$. This may be useful in terms of the growth accounting approach to highlight possible measurement errors. However, the accounting identity must hold and so either r_t or J_t must be adjusted to ensure that the identity is consistent.

It is important to mention that, in his calculations, Hsieh did not use the accounting identity exactly as described above, i.e., by calculating residually one of the five variables. Hsieh did not calculate the five series independently; instead of calculating *r* residually, he computed the *rental price of capital* (v) as $v_t = q_t [\rho_t + \delta_t - \dot{q}_t]$ (Jorgenson 1963) where *q* is the price of capital, ρ is a measure of the cost of capital, δ is the depreciation rate and $\dot{q}$ is the capital gain or loss. If this

---

5 Consider the following. From the NIPA we can obtain the labor share as $a_t \equiv (\mathrm{W_t} / V_t)$. Now suppose we obtain independent data on the average wage rate (*w*) and employment (*L*), and find that $a_t \neq (\mathrm{w}_t L_t / V_t)$. However this identity is calculated, the four series have to preserve it. The same applies to the variables that make up the capital share.

is used in the identity with the other variables independently measured, then there may be a statistical or measurement problem along the lines outlined above. Nevertheless, in Felipe and McCombie (2005a, 2005b) we show that this issue does not pose a conceptual problem for our argument.[6]

Before concluding this analysis of the Barro-Hsieh approach, it is worth noting that, as indicated above, a crucial assumption underlying this whole approach is that a well-behaved aggregate production function exists. This is taken to be self-evident and is rarely, if ever, explicitly discussed. The production function is a relationship between the amount of output produced and the quantities of inputs used (in a context of optimal allocation), that is, $Q = A(t)F(L,K)$, where Q, L and K are output, labor and capital, respectively, all measured in physical terms.

However, this is not an innocuous assumption and indeed there are two theoretical reasons for believing that the aggregate production function cannot exist. The first stems from the results of the Cambridge Capital Theory Controversies, which seem to be long forgotten and rarely mentioned in the literature.[7] Nevertheless, however inconvenient for the aggregate neoclassical approach, these results still stand. See, for example, the recent retrospective view of Cohen and Harcourt (2003). Secondly, there are the aggregation problems, recently surveyed by Felipe and Fisher (2003). The most important results of this literature can be summarized as follows:

---

[6] Hsieh did not use, and hence did not calculate, all the series in the identity as he concentrated on the dual measure of TFPG. We thank Hsieh for pointing this out in correspondence with him. However, when he calculated the return to capital derived from the national accounts, implicitly he was using all the series in the identity and calculated one of them (the return to capital) residually.

[7] Bernanke (1987), in passing, while discussing an empirical paper by Romer where the latter tested a series of endogenous growth models, made the following remark: "It would be useful, for example, to think a bit about the meaning of those artificial constructs "output," "capital," and "labor," when they are measured over such long time periods (*the Cambridge-Cambridge debate and all that*)" (Bernanke 1987, p.203; italics added). Sylos-Labini (1995) recently wrote: "It is worth recalling these criticisms, since an increasing number of young and talented economists do not know them, or do not take them seriously, and continue to work out variants of the aggregate production function and include, in addition to technical progress, other phenomena, for example, human capital" (Sylos-Labini 1995, p.487).

- Except under constant returns, aggregate production functions are unlikely to exist at all.
- Even under constant returns, the conditions for aggregation are so very stringent as to make the existence of aggregate production functions in real economies a non-event. This is true not only for the existence of an aggregate capital stock but also for the existence of such constructs as aggregate labor or even aggregate output.
- One cannot escape the force of these results by arguing that aggregate production functions are only approximations. While, over some restricted range of the data, approximations may appear to fit, good approximations to the true underlying technical relations require close approximation to the stringent aggregation conditions, and this is not a sensible thing to suppose.

Suffice to say here that both these critiques addressed, from very different points of view, the question of whether or not the aggregate production function, with the usual neoclassical properties, is a legitimate concept, even when viewed as an approximation. Both criticisms come to the same conclusion on this point: the existence –and hence the use- of aggregate production functions is very problematic. This implies that the connection made in neoclassical macroeconomics between the identity, the aggregate production function and Euler's theorem is very tenuous, to say the least. Indeed, this line of reasoning is untenable if the aggregate production function does not exist;[8] and as Fisher pointed out, "If aggregate capital does not exist, then of course one cannot believe in the marginal productivity of *aggregate* capital" (Fisher 1971a, p.405; italics in the original).

Why is then the aggregate production function still so widely used? The answer, although it is rarely explicitly stated, must be that ever since the early estimations of Douglas and his colleagues, the statistical estimation of aggregate production functions gives good fits with the estimates of the output elasticities close

[8] As a consequence, parameters such as the elasticity of substitution are meaningless. In the words of Fisher et al. (1977): "the elasticity of substitution in these production functions is an "estimate" of nothing; there is no true aggregate parameter to which it corresponds" (Fisher et al. 1977, p.312).

to the observed factor shares, although time-series results are known for their fragility (see Sylos-Labini 1995; see Felipe and Fisher 2003 for a detailed discussion). This argument relies on Friedman's (1953) instrumental methodological justification that what matters are the successful predictions of a theory, regardless of the unrealism of the underlying assumptions. Not only is this view no longer widely held by philosophers of science, but our argument shows that this justification is flawed, even on its own terms (McCombie 1998a).

3. ESTIMATING THE AGGREGATE PRODUCTION FUNCTION

Barro (1999) briefly compared growth accounting to the econometric estimation of the production function and favored the former method. He argued that growth accounting is preferable because the latter has some important disadvantages (Barro 1999, pp.122-123): (i) the growth rates of capital and labor are not exogenous variables with respect to the growth of output; (ii) the growth of capital is usually measured with error. This often leads to low estimates of the contribution of capital accumulation; and (iii) the regression framework must be extended to allow for variations in factor shares and the TFP growth rate.

It is useful to address the last point first since the arguments concerning it render the first two largely irrelevant. For this purpose, let us return to equation (2) and assume that in this economy factor shares are constant, for example, because firms pursue a constant mark-up pricing policy.[9] Suppose also that in this economy wage and profit rates grow at constant rates.[10] Then, this expression becomes:

$$\hat{V}_t \equiv a\hat{w} + (1-a)\hat{r} + a\hat{L}_t + (1-a)\hat{J}_t$$

[9] The constancy of the factor shares does not imply a Cobb-Douglas production function, as Fisher's (1971b) simulation exercises showed.

[10] The "stylized" facts are that the trend growth of the real wage rate is constant, while the rate of profit shows no systematic trend. The rate of profit, and to a lesser extent the real wage rate, exhibit procyclical variation. The constancy of factor shares is a further stylized fact.

$$\equiv \lambda + a\hat{L}_t + (1-a)\hat{J}_t \qquad (6)$$

If these assumptions hold, equation (6) will continue being the accounting identity in growth rates. It implies nothing about the technological conditions of production. Integrating equation (6) and taking anti-logarithms gives:

$$V_t \equiv A_0\, exp(\lambda\, t)\, L_t^{a}\ J_t^{1-a} \qquad (7)$$

where $\lambda \equiv a\hat{w} + (1-a)\hat{r}$ and A_0 is the constant of integration. It should be noted that this is merely another way of writing the identity, although in this case with the two assumptions. This result indicates that if one gets data for the economy in question and regresses output (*V*) on labor (*L*), capital (*J*), and an exponential time trend (*t*), i.e., $V_t = B_0\, exp(b_1\, t)\, L_t^{b_2}\ J_t^{b_3}\ exp(\varepsilon_t)$, where ε_t is the random disturbance, and the two assumptions made happen to be correct, it is obvious, by comparison with (7), that the fit will be perfect, $b_1 \equiv a\hat{w} + (1-a)\hat{r} \equiv \lambda$, $b_2 \equiv a$, and $b_3 \equiv 1-a$. Under a neoclassical interpretation, the equality of the elasticities to the factor shares would be interpreted as validation of the neoclassical theory of factor pricing and, consequently, that markets are competitive. Moreover, the result indicates "constant returns to scale." But this economy could well be a country with a command economy where factors are not paid their marginal products. All we have used in deriving equation (7), to paraphrase Hsieh and Barro, is the "condition" that output equals the payment to the factors of production, together with the two stylized facts. The fact that the estimated "output elasticities" closely approximate the factor shares does not imply that markets are competitive, and that there are constant returns to scale. This correspondence merely follows from the accounting identity.

What if the assumptions about the factor shares and the wage and profit rates are incorrect (and this addresses Barro's third point)? The estimation of either equations (6) or (7) will, in these circumstances, give a poor statistical fit with the estimated coefficients with large standard errors and diverging significantly from the factor shares. But this problem is not insurmountable. For most economies factor shares are sufficiently constant so that the estimation of (6) or (7) will still yield remarkably good results. But even if this were not the case, all that would be needed is to specify a function that tracks the shares correctly, then substitute into (2) and proceed as above. Finding this function might be more or less complicated empirically, but conceptually the issue is straightforward.[11]

The second assumption to derive equation (7) is the one about the constancy of the growth rates of the wage and profit rates. Empirically, it is this one that causes most problems in the estimation of production functions using time-series data. From the identity, we know that $TFPG_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t$. Plots of the Solow residual typically show a procyclical fluctuation around its mean growth rate (see, for example, Solow, 1957). Consequently, in spite of the strong trend underlying the growth rate, using a linear time trend (in the log-levels specification) or a constant (in the growth rate specification) does not accurately explain the variation in this variable. In fact, the problem in the regression is akin to one of omitted variable bias. For all practical purposes, $TFPG_t$ is omitted from the regression, thus biasing the estimates of capital and labor. This is the problem to which Barro refers. What is the solution? Given the typical path of TFPG, a complex trigonometric function, rather than a simple time trend, will do a much better job at tracking the path of $TFPG_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t$. But once this is done, we will get back to the identity, elasticities will equal the factor shares, and we will find putative constant returns to scale. Summing up: if the production function is estimated "correctly" (i.e., if the functional form chosen correctly approximates the accounting identity), no data set can refute the null hypotheses that the elasticities equal the factor shares and constant returns to scale. It is not possible to test the existence of an aggregate production function.[12]

---

[11] One could, for example, try a Box-Cox transformation, which is analogous to a CES "production function".

[12] The problem is a little more complicated than this in that by judicious choice of a complex time-trend one could get a perfect statistical fit with any estimated value of the "output elasticities" that one likes. To see this, consider $\hat{D} = \hat{V} - \phi(\hat{J} - \hat{L})$ where $0 \le \phi \le 1$. If a

There is a different solution to the conundrum that Barro hints at in his second point, when he mentions that "the capital stock is unlikely to correspond to the stock currently utilized in production" (Barro 1999, p.123). He refers to the use of a capital stock corrected for utilization. This does not matter for the accounting identity, but could potentially solve the "problem" of estimating the production function. The issue is as follows: as we have noted, if one looks at the components of $TFPG_t \equiv a_t \hat{w}_t + (1 - a_t)\hat{r}_t$, the one that mainly varies is $\hat{r}_t$, and it does so procyclically. As we are arguing that this variable is being omitted from the regression, the inclusion of any other variable that moves procyclically, namely the adjusted for capacity utilization capital stock, will serve as a proxy and will do a good job. If capital's share is roughly constant, imparting a procyclical fluctuation in the capital stock will reduce that exhibited by the rate of profit. Barro is right, but for the wrong reason.

The answers to Barro's first and second points on why growth accounting is preferable to the estimation of the production function follow from the above discussion. It does not matter whether the growth of capital and labor are exogenous or endogenous, if all that is being estimated is an identity. Moreover, at the theoretical level, these constructs do not satisfy Fisher's aggregation conditions, so it is dubious what empirical relevance they have. Econometrically, the issue is not one of instrumental variables or unit roots since, once again, we are dealing with an identity. Finally, the possibility of measurement error is potentially a serious issue, but it will affect both econometric estimation and growth accounting.

4. SOME SIMULATION EXPERIMENTS

To illustrate the above arguments, we undertook some simple simulations with a view to studying what growth accounting is actually measuring. The key question behind the

---

complex time trend can be found that gives a near perfect fit to $D(t)$ in the equation $lnV = lnD(t) + b_4 ln(J - L)$, then the estimation of this Cobb-Douglas equation will give $b_4 = \phi$. We have no way of deciding between any of the specifications, as, by definition, they all give perfect fits.

exercise is the following: if an aggregate production function does not exist, what is being measured as productivity, in particular as total factor productivity growth, in exercises that use aggregate data? This has the following implications: (i) if the answer is that some average of individual firms' productivities then, does the aggregate method still yield such an average? If not, why not?; and (ii) more generally, how and to what extent are we misled by the results of aggregate growth accounting? It should be emphasized that we are not performing Monte Carlo simulations. The purpose of this exercise is merely to generate a data set whose underlying structure is known, in both physical and value terms in order to highlight the different results that the two types of data lead to in growth accounting exercises. Monte Carlo simulations studying related issues, but answering different questions, were performed by Felipe and Holz (2001). See also Fisher (1971b) and Shaikh (1980).

Table 1 summarizes the characteristics of the simulations. We assume that there are well-defined micro-production functions, which are specified in physical terms, as ideally they should be. The constant price value of output was calculated through a mark-up and the capital stocks were generated residually through the NIPA identity, equation (1). It is assumed that each firm produces a homogenous output, which may or may not be the same for all firms. The analysis does not depend on this assumption so long as, in the former case, it is not possible to recover the physical quantities from the value data. In the latter case, i.e., output is not homogeneous across firms, we cannot, of course, estimate a cross-section production function using physical data. It is important to emphasize that we generate two types of data, in physical terms and in value terms. The former are assumed and generated so as to give a good fit to the Cobb-Douglas function. The investigator does not know this. Value data, as indicated above, are generated through the accounting identity. The investigator knows all the value data but cannot recover the physical data from them.

It should also be noted that in the case where a cross-firm production function is estimated, no aggregation problem à la Fisher is involved. This is an important point. If we need to estimate a production function using outputs and inputs summed over different firms, we encounter all the well-known aggregation problems. As value data has to be used in estimating this aggregate production function, we can explain why regressions using these data give a good fit to the aggregate data when theoretically they should not (Fisher, 1971b, Felipe and McCombie, 2005c). But the problem is even more fundamental than this. As will be demonstrated, the accounting identity presents insurmountable problems of interpretation, even when there are no aggregation problems of any kind regarding functional forms, or affecting output, labor or capital (see Felipe and Fisher 2003) or problems of the type discussed in the context of the Cambridge debates regarding the nature and construction of capital stocks (Cohen and Harcourt 2003).

The important aspect of our simulations is that they show how the use of value data can give results at variance with the true magnitudes of the underlying production functions and, therefore, misleading numerical estimates of both the parameters of the production function and of the "rate of technical progress." For clarity purposes, we will confine the term "technical progress" to that calculated using physical data; and the term "total factor productivity" to that calculated using value data.

Table 1. SUMMARY OF THE CHARACTERISTICS OF THE SIMULATIONS

* 10 firms, one period

* Identical $Q_i = (A_0 L_i)^{\alpha} K_i^{(1-\alpha)}$ Q_i and K_i are generates as random variables. L_i is generated through the production function. These are physical data.

* Q_i is assumed to differ across firms; K_i is specific to the firm; A_0 is the same across firms and normalized to 1

* Elasticites: $\alpha = 0.25$; $1-\alpha = 0.75$ with a random error to avoid multicollinearity

* Value data: Firms set prices as a mark-up on unit labor costs, i.e., $p_i = (1+\mu) w_i L_i / Q_i$, where $\mu = 0.33$ is the same across firms

* Wage rate: $w_i = w$ is the same across firms

* Profit rate: $r_i = r = 0.10$ is the same across firms

* Output in value terms: $V_i = p_i Q_i$

* Capital stock in value terms: $J_i = (V_i - w_i L_i) / r_i = (V_i - wL_i) / 0.1$

* Labor share (value terms): $a_i = (wL_i) / V_i = 1/(1+\mu) = 0.75$; Capital share: $(1-a_i) = \mu/(1+\mu) = 0.25$; for each firm, Mean of a_i is 0.744 (range 0.698 - 0.795)

Rate of technical progress and TFPG

* Output of the 10 firms grows at different rates, but $\hat{Q}_i = \hat{K}_i$

* Same rate of technical progress, $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$, assumed to be $0.5\% = 0.25(\hat{Q}_i - \hat{L}_i)$

* Growth of employment $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$

* Elasticities (physical terms): $\alpha = 0.25$; $1-\alpha = 0.75$

* Average shares (value terms) are as above

* True rate of technical progress (firm level): $\varphi_i = \hat{Q}_i - 0.25\hat{L}_i - 0.75\hat{K}_i$

* TFPG: $TFPG_i = \hat{V}_i - 0.75\hat{L}_i - 0.75\hat{J}_i$

Increasing Returns to Scale

* $\hat{Q}$, $\hat{L}$, $\hat{K}$ as before

* Elasticities $\alpha = 0.3$, $1-\alpha = 0.9$. Degree of returns to scale = 1.2

* Value data calculated as before and mark-up μ is also 0.33.

(i) Cross-Firm Estimation of the Production Function

In the first example, data in physical units were generated for 10 firms for one period under the assumption that they all have identical Cobb-Douglas constant-returns-to-scale production functions given by

$$Q_i = (A_0 L_i)^{\alpha} K_i^{(1-\alpha)} \qquad (8)$$

where Q_i is the number of units of homogeneous output, generated as a random variable; K_i is the number of identical machines which are specific to the particular industry, also generated as a random variable; L_i is the level of labor input, generated through the production function; A_0 takes the same value for all firms and was normalized to unity. The parameters α and $(1-\alpha)$ are the output elasticities of labor and capital, respectively, and are constructed to take values of 0.250 and 0.750.[13] The output elasticities were deliberately chosen to be the converse of the factor shares found in the NIPA.

In order to generate the monetary values, each firm sets prices as a mark-up on unit labor costs, i.e.,

$$p_i = (1 + \mu) w_i L_i / Q_i \qquad (9)$$

The mark-up (μ) is the same for all firms and takes a value of one third, so $(1+\mu)$ = 1.333. The wage rate is the same across firms, the same as the profit rate *r*, which takes a value of 0.10. The *value* of the capital stock was calculated residually through the accounting identity as $J_i \equiv (V_i - wL_i)/r$, where V_i is value added, constructed as $V_i = p_i Q_i$ for each firm using equation (9). The values of the factor shares are directly calculated using these value data. Labor's share is calculated as

13 To prevent perfect multicollinearity, a small random variable was added to these and, where necessary, other variables used in the simulation.

$a_i = (wL_i)/V_i$ and capital's share as $(1-a_i)$. It should also be noted that $a_i = 1/(1+\mu)$, and so it takes a value of 0.75 for each firm, with a small variation due to the error term added. The mean value of labor's share for the10 firms is 0.744 (with a range of 0.698-0.795.

The mean value of the capital-output ratios in value terms (J_i / V_i) is 2.57 with a range of 2.24 to 3.18. These values are very close to what are observed empirically, and are the result of a roughly constant rate of profit and constant factor shares. As $J/V = (1-a)/r$, where $(1-a)$ is capital's share and, as noted above, is approximately equal to 0.25 and the profit rate is 0.10, the capital-output ratio will not differ much from 2.50. As we are dealing with individual firms and we design the simulations, we know both the physical data and the values, since we know the prices. But let us assume that the prices are unknown to the researcher, as is usually the case, because the output and capital stocks for different firms are aggregated in the NIPA using value measures. Consequently, V and J (in constant prices, although since we only have one period, the distinction between current and constant prices does not arise) were taken as proxies for Q and K.

These value data were then used to estimate a cross-firm production function. The results of the estimation are:

$$\ln V = \underset{(478.77)}{2.867} + \underset{(136.40)}{0.750}\ln L + \underset{(45.41)}{0.250}\ln J \qquad \bar{R}^2 = 0.9999 \quad \text{s.e.r.} = 0.0025$$

This gives a remarkably close fit to the Cobb-Douglas production function, which is to be expected given the method used to construct the data. However, some of Douglas and his colleagues' early studies, which used real, as opposed to simulated, cross-state data, also found very close statistical fits.[14] The sum of the estimated

[14] For example, Douglas (1976, p.906) reports the following results for a production function based on American cross-section studies, 1904, 1909, 1914 and 1919.

Year	α	β	$\alpha + \beta$

coefficients is 1.00 and this is not significantly different from unity (the value of the *t-ratio* testing this hypothesis is 0.02). With the close correspondence between the supposed "output elasticities" and factor shares calculated from the data (0.750; 0.744 and 0.250; 0.256), it is little wonder that such results could be interpreted as providing evidence in favor of competitive markets and disproving the Marxian argument, as Douglas (1976, p.914) claimed.

This is not withstanding the fact that factors are *not* paid their marginal products in physical terms in our simulation data. Competition could force firms to be *x*-efficient so that firms do hire the factors of production up to the point where their physical returns equal their factor rewards in terms of the commodity produced. This would determine the optimal L_i / Q_i, which is used in the mark-up pricing equation. However, using value data would still give estimates of the "output elasticities" equal to $1/(1+\mu)$ and $\mu/(1+\mu)$, respectively.

However, it should be emphasized that the estimated "output elasticities" are, of course, not the same as the "true" output elasticities of the micro production function. In other words, the true output elasticity of labor is 0.25, but the estimate using value data is 0.75.

The goodness of fit is dependent upon the degree of variation in the mark-up. With identical mark-ups, the fit is exact (and estimation is not possible because of perfect multicollinearity). Indeed, it is the constant mark-up that is solely responsible for generating the "spurious" Cobb-Douglas. To demonstrate this, the physical values of the three series *Q*, *L* and *K* were next generated as random numbers. *V* and *J* were

1904	0.65 (32.5)	0.31 (15.5)	0.96
1909	0.63 (31.5)	0.34 (17.0)	0.97
1914	0.61 (30.5)	0.37 (18.5)	0.98
1919	0.76 (38.0)	0.25 (12.5)	1.01
Average	0.66 (33.0)	0.32 (16.0)	0.98

Notes: *t-value*s in parentheses.
Total number of observations: 1490.

calculated as before. The estimation yielded a very good fit to the Cobb-Douglas with the values of the "output elasticities" the same as before (the result is not reported here). This does not necessarily mean that we are postulating that that output is actually a random function of the inputs. However, when one considers the complex production processes of any modern firm, there may be some individual parts of the process subject to fixed coefficients, whereas others subject to differing elasticities of substitution, to say nothing of differences between plants in managerial and technical efficiencies. Thus, the "randomness" may simply be a reflection of the severe misspecification error inherent in specifying the micro-production function as a Cobb-Douglas. But the important point to note is that even in this case, where there is no well defined micro- production function, the use value added data will give the impression that there exists a well-behaved aggregate Cobb-Douglas production function.

(ii) Rate of Technological Progress and Total Factor Productivity Growth

In order to calculate the growth of total factor productivity, we need the growth rates of output, capital and labor. We assumed that output of the 10 hypothetical firms grows at different rates (we only have one single period), but, for expositional purposes only, the series were constructed such that the growth rate of the physical capital-output ratio is zero (i.e., output and capital grow at the same rate) for all firms. It was also assumed that each firm experiences the same rate of technical progress (φ_i), 0.5% per annum, equal to $\varphi_i = \alpha(\hat{Q}_i - \hat{L}_i)$. This is due to the fact that the underlying production functions are Cobb-Douglas and that the growth of output equals the growth rate of capital. Hence, the growth rate of employment for each firm was constructed as $\hat{L}_i = \hat{Q}_i - (\varphi_i / \alpha)$, where $\varphi_i = 0.5\%$, as noted above. The output elasticities of labor and capital in *physical terms* are again 0.25 and 0.75, the average *value shares* are 0.745 (with a range from 0.698 to 0.795) and 0.255, and the aggregate shares are also 0.745 and 0.255 since each firm has the same mark-up, which means that the labor share of each firm is the same and if we aggregate over firms, the aggregate share comes to be about the same.

The rate of technical progress using the physical simulated data would be calculated by the investigator unaware of its value (although *we* know it by construction, 0.5%) for each hypothetical firm separately using the standard growth accounting equation, that is,

$$\varphi_i \equiv \hat{Q}_i - a\hat{L}_i - (1-a)\hat{K}_i \qquad (10)$$

where a and $(1-a)$, the factor shares of labor and capital, are assumed to equal the output elasticities α and $(1-\alpha)$, 0.25 and 0.75, respectively.

As the rate of technical progress is the same for each firm, we can talk about the rate of technical progress being 0.5% per annum; even in the case where we assume that the physical outputs of the various firms are not homogeneous.

However, let us assume, once again, that the individual prices of the various firms are not available and so it is not possible to extract data on the physical units of output. All that can be used in empirical work, as is usually true in practice, is the constant price value of output and of the capital stock. The growth of total factor productivity is given by:

$$TFPG_i \equiv \hat{V}_i - a\hat{L}_i - (1-a)\hat{J}_i \qquad (11)$$

where now the shares are 0.75 and 0.25, respectively.

The unweighted mean rate of total factor productivity growth of the individual firms is 1.49% per annum, which, not surprisingly, is almost identical to the rate of total factor productivity growth obtained by aggregating the value data over all 10 firms and using these in equation (11) (1.48% per annum).

Thus, the use of physical data yields technical progress accounting, on average (the unweighted mean), for $\varphi_i = 0.25(\hat{Q}_i - \hat{L}_i)$, that is, 25% of labor productivity growth, with a very small difference between firms due to the small random element introduced for the reasons noted above.[15] On the other hand, the use of value data for each of the 10 firms gives a mean value of the rate of total factor productivity that is $TFPG_i = 0.75(\hat{V}_i - \hat{L}_i)$, or 75% percent the growth of labor productivity, with a range from 80 per cent to 70 per cent. And the figure using the aggregate data (i.e., using the aggregate values of output, labor and capital) is 74 percent. The reason for the marked difference between these values and the "true" rate of technical progress is that labor's share of output in value terms is 0.75, while the "true" output elasticity of the firms' production functions is 0.25.

It is worth noting that *TFPG* in equations (2) and (3) can be written as $TFPG_t \equiv \hat{V}_t - a_t \hat{L}_t - (1 - a_t)\hat{J}_t \equiv a_t \hat{q}_t + (1 - a_t)\hat{j}_t$, where $\hat{q}_t = \hat{V}_t - \hat{L}_t$ denotes the growth rate of labor productivity and $\hat{j}_t = \hat{V}_t - \hat{J}_t$ is the growth rate of capital productivity. This is a weighted average of the growth rates of labor and capital productivity. Therefore, it could be argued that *TFPG* is an aggregate measure of productivity growth. This interpretation faces, however, the problems discussed in this subsection, namely, that the figure computed is not equivalent to the true rate of technical progress.

Consequently, the use of value data produces a significantly different estimate of the rate of technical progress, compared with the "true" value obtained using physical data. Even with well-defined underlying Cobb-Douglas production functions expressed in physical terms, the use of value data as a proxy for output can give very misleading estimates of the rate of "technical progress".

(iii) Increasing returns to scale and total factor productivity growth.

What happens if the individual firms are subject to increasing returns to scale when physical data are used? To examine this question, we first estimated the cross-firm production functions using value data when the micro production functions exhibit the

[15] This is because there is no growth in the physical capital-output ratio, $\varphi_i \equiv \alpha(\hat{Q}_i - \hat{L}_i)$ and α, the physical output elasticity of labor, is equal to 0.25. Hence the rate of technical progress equals one quarter of the growth of labor productivity.

same degree of increasing returns to scale. The data for the inputs in physical terms were the same as those used in the previous simulation, with the exception that now the elasticities were multiplied by 1.20, so $\alpha^{'}$ = 0.30 and $\beta^{'}$ = 0.90. This represents a substantial degree of returns to scale and results in a value of output that is significantly larger than when constant returns to scale are imposed. The value data were calculated the same way as before, with a mark-up once again of 1/3.

Estimating the unrestricted Cobb-Douglas production function gives a result that is virtually identical to that for constant returns to scale, and reported above, except for a change in the value of the intercept. Consequently, we do not report the results here. The estimates of the putative output elasticities are once again very close to the observed (value) factor shares and sum to unity, thereby erroneously suggesting that the production process of the various firms are subject to constant returns to scale. The reason for this seemingly paradoxical result is that the calculation of value added is given by $pQ = V = (1 + \mu)wL$ and as nominal wages and the level of employment are the same as before, so is the constant price measure of value added, although the price per unit is now lower (there is no inflation in the simulated data). Recall that section 3 showed that with value data, estimation of the production function would yield elasticities equal to the factor shares.

Next, we calculated the rate of technical progress and the growth of total factor productivity. For comparability with the constant returns to scale case, the growth rates of physical output, capital and labor were the same as before. The rate of technical progress was calculated using the physical data as

$$\varphi_i = \hat{Q}_i - \alpha^{'}\hat{L}_i - \beta^{'}\hat{K}_i \qquad (12)$$

where $\alpha^{'} = 1.2\alpha$ and $\beta^{'} = 1.2(1 - \alpha)$. It can be seen that the rate of technical progress calculated using equation (12) will differ from the 0.5% in the case of constant returns to scale. In fact, it will be on average lower, given the larger weights of the growth of the factor inputs. The rate of technical progress, calculated using equation (12) for each firm now varies considerably across firms (for reasons of space we do not report the full results).[16] The unweighted mean is (coincidentally) 0.00 per cent per annum, with a range of ± 0.4 percentage points per annum.[17]

On the other hand, the growth rates of total factor productivity of the individual industries, calculated using equation (11) and value data, are again all approximately 1.5% per annum. This is because the shares of labor and capital in value terms are once again 0.75 and 0.25, and the growth rates of V, L and J are the same as before.[18] Thus, the use of value data can give a very misleading estimate of the true rate of technical progress. The use of value data erroneously ascribes the effect of increasing returns on increasing the efficiency of the factors of production to the rate of technical progress.

(iv) Summary

Of course, the actual figures are arbitrary as they are dependent upon the assumed rates of growth of physical output, capital and employment. Nevertheless, we can draw some conclusions from these simulations.

- The growth of total factor productivity depends crucially on the weights attached to the growth of capital and labor. The growth accounting approach assumes that factors are paid their marginal products and hence the technologically determined output elasticities will equal the factor shares. However, when value data is used we have shown that the factor shares will *always* equal the putative output elasticities and both are determined by $1/(1+\mu)$ (labor) and $\mu/(1+\mu)$ (capital), where $1+\mu$., it will be recalled, is the mark-up. The estimates of the output elasticities using value data will almost

16 This is because while the growth rates of *Q* and *K* are the same between firms, employment growth rates differ and so the change in weighting causes the rate of technical change now to differ across firms.

17 As we cannot sum across the physical quantities, we cannot calculate a meaningful average rate of technical progress, as the individual rates cannot be unambiguously or uniquely weighted. Nevertheless, we did calculate the unweighted mean.

18 With a constant mark-up of 1.33, the shares will be always 0.75 and 0.25, regardless of the technical conditions of production (e.g., the degree of returns to scale).

certainly differ from the true ones (always assuming that there is a well-defined micro-production function in physical terms).

- Where it is possible to compare the "true" growth rate of technical progress with the growth of total factor productivity in value terms (namely at the firm level here), the two values will probably differ markedly. In general, it is not possible to recover the physical quantities (of both output and capital) from the value data through the individual prices, and so resort is made to value data with potentially very misleading results. Where the physical data can be inferred, it can only be done at a very low level of aggregation. It requires each output and capital good to be measured separately in physical terms.

- The problems posed by the accounting identity are independent from, and in a sense more fundamental than, either the aggregation problem or the Cambridge Capital Theory Controversies.

In these simulations, we have found that there is a very close statistical fit to the estimation of the production function. However, with time-series data, as opposed to cross-section data, the standard errors are often much larger and the coefficient of the capital stock often has the wrong sign, i.e., negative. The reason for this is straightforward. We know that the accounting identity $\sum V_{it} \equiv \sum w_{it} L_{it} + \sum r_{it} J_{it} \equiv w_t L_t + r_t J_t$, where V_t, L_t , and J_t are the aggregate values and w_t and r_t are the average values, must hold. If there is a constant mark up (and hence aggregate constant shares), differentiating the identity and then integrating gives

$$V_t = B_0 w_t^a r_t^{(1-a)} L_t^a J_t^{(1-a)} \qquad (13)$$

In logarithmic form this becomes

$$lnV_t = ln B_0 + [a ln w_t + (1-a) ln r_t] + a ln L_t + (1-a) ln J_t \qquad (14)$$

Nevertheless, in nearly all estimations of production functions, the expression in brackets (i.e., the weighted sum of the logarithms of the wage and the profit rates) is proxied by a linear time trend. However, in practice, this term shows a pronounced cyclical fluctuation and so the use of a linear time trend often induces a significant misspecification. It should also be noted that in the neoclassical schema, where it is implicitly assumed that physical quantities can be adequately proxied by value terms, if perfect competition prevails and factors (at the aggregate level) are paid their marginal products, then from the dual, $TFPG_t = a\hat{w}_t + (1-a)\hat{r}_t$.

Consequently, there is no theoretical justification for imposing a linear time trend to capture the putative rate of technical progress. Cobb and Douglas (1928) in their seminal paper did use time-series data and coincidentally obtained a close statistical fit with factor shares equal to the “output elasticities” – it was coincidental because the results were not robust; dropping the last few observations radically altered the estimates and no time trend was included. (McCombie 1998b; Felipe and Adams 2005). It was recognized early on in the 1930s that the use time-series data often gave implausible results and this may be the reason why Douglas and his colleagues changed to the almost exclusive use of cross-state and cross-industry data in their numerous estimations of production functions during the 1930s.[19]

5. A REINTERPRETATION OF GROWTH ACCOUNTING EXERCISES

It should be clear by now that all estimations of aggregate production functions do is to approximate the NIPA accounting identity, and as such they are exercises which, although algebraically correct, they stand without theoretical foundation. What about

---

19 The cross-sectional Cobb-Douglas is equivalent to $V_i = B_0 w_i^a r_i^{(1-a)} L_i^a J_i^{(1-a)}$. The term $w_i^a r_i^{(1-a)}$ shows relatively little variation when the data is drawn from the same country for firms, industries or states.

growth accounting? Our argument above was that the procedure Hsieh and Barro proposed has an alleged rationale because they believe that an aggregate production function and its associated marginal productivity theory of factor pricing are behind the accounting identity. But we have argued that this completely neglects the fact that aggregate production functions do not exist due to the aggregation problem; and that the "production function theory" cannot be tested because it cannot be refuted, i.e., it is possible to find a specification of the identity that will yield a very high fit, potentially unity, and indicate that markets are competitive and constant returns. Moreover, the simulations have shown that the results of growth accounting exercises with aggregate data do not yield the true rate of technical progress.

This indicates that the accounting identity stands on its own. There is no theory behind it. So, what does the transformation of an accounting identity tell us about the economy? We have concluded that $TFPG_t \equiv V_t - a_t \hat{L}_t - (1-a_t)\hat{J}_t$ does not measure the true rate of technological progress. Can $TFPG_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t$ be interpreted as a measure of the change in technical progress the way it is done in neoclassical growth accounting exercises? We do not think so.

Note, first, that it is straightforward to derive φ_t from the income accounting identity expressed in nominal and real terms. By expressing both identities in growth rates and subtracting them, one can derive an expression for the rate of cost diminution ($\hat{c}_t^*$), or the rate of change in unit costs of production (the so-called dual) as:

$$\hat{c}_t^* = \hat{P}_t - [a_t \hat{w}_t^* + (1-a_t)\hat{r}_t^*] = -[a_t \hat{w}_t + (1-a_t)\hat{r}_t] = -TFPG_t \qquad (15)$$

where $\hat{P}_t$ is the growth of the implicit price deflator, $\hat{w}_t^*$ is the growth rate of the nominal wage rate, and $\hat{r}_t^*$ is the growth rate of the nominal profit rate. What can we make, therefore, of growth account exercises? We believe that $TFPG_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t$ can be interpreted *only* as a *measure of distributional changes* (not in a zero-sum sense).

What then are we to make of Barro's (1999, p.123) argument that an economy that experiences an increase in both its real wage and profit rates must have increased its overall level of productivity? It could be argued that $TFPG_t = a_t \hat{w}_t + (1 - a_t)\hat{r}_t$ measures such a rate of growth of efficiency. Certainly, under these circumstances one can say that the economy is somewhat better off, since obviously this is contributing positively to output growth. The point to note is that it is not possible to ascribe this unambiguously to the result of technical change the way it is done in the neoclassical model (i.e., by claiming that this idea derives from a testable model). There is no reason to assume that factor shares, i.e., the "appropriate" or theoretically justified weights according to Barro (1999, p.123), equal the output elasticities of the true aggregate production function (if it, in fact, exists), or that production is necessarily subject to constant returns to scale (although this is what the use of value data will show). The derivation is simply a tautology resulting from an identity with no behavioral assumptions or implications. The wage rate is likely to be correlated with labor productivity in value terms, and changes in the rate of profit are also likely to be associated with changes in the capital-output ratio, also measured in value terms.

Consequently, $TFPG_t = a_t \hat{q}_t + (1 - a_t)\hat{j}_t$ and given little growth in the capital-output ratio, this becomes $TFPG_t = a_t \hat{q}_t$. But this result follows directly from the mark up, rather than the underlying technology. The only possible way to argue that $TFPG_t$ in equation (2) is a measure of the rate of technical change is to postulate the existence of an aggregate production function or cost function, together with the conditions for producer equilibrium. This is required as a justification for using the factor shares to weight the growth rates of the wage and profit rates in order to derive a combined index of total factor productivity growth, and for considering $a_t \hat{L}_t$ and $(1 - a_t)\hat{J}_t$ as a measure of the contribution, in a *causal* sense, of the growth of the factor inputs to output growth.

Suppose, for the sake of the argument, that the factor shares are determined by the relative bargaining power of labor *vis-à-vis* capital. Let us assume further that capital's share increases and labor's share falls due to changes in legislation relating to trade unions. Even though in physical terms the growth rates of output and inputs, and hence of technical change, remain the same by assumption, the calculation of TFPG will show a decline. Hence, the concept of total factor productivity growth cannot be given purely a technological interpretation.

Hart (1996) also starts from the accounting identity and emphasizes, like us, that "accounting identities, unlike the usual econometric estimates of production functions, hold in equilibrium and in disequilibrium" (Hart 1996, p.226). He views the identity as representing the trading accounts of the representative firm, but because of data limitations, he uses macroeconomic data. However, he calculates TFPG, or multifactor productivity (MFP) growth, as he terms it, as the weighted growth of factor prices, citing in support of this the discussion of the dual by Jorgensen and Griliches (1967). Nevertheless, somewhat confusingly, he argues that the accounting identity (dual) approach makes it easier to explain the effects on MFP of "disequilibrium, market imperfections, trade-union cost pushes, OPEC oil-price squeezes, Government price controls and incomes policies, regulation and deregulation and of other forces which influence profit margins. It must be remembered that these dual measures of MFP are purely statistical, and, so far no use has been made of production functions, returns to scale, marginal productivity, competitive equilibrium conditions or any economic theory of production" (Hart 1996, p.228).

What is the justification for using factor shares as weights for the growth of the factor inputs, as Hart does? The standard justification rests solely on the argument that they are a good approximation to the output elasticities. However, this requires standard neoclassical production theory. Hart (1996, p.229) gives an example where an increase in V goes entirely to labor due to trade union bargaining: "so the direction of causation is from left to right", i.e. from the increase in V to the increase in w. This means that that the share of wages increases, but this may have nothing to do with the output elasticity of labor or with a change in the technical conditions of production.[20] Thus, the shares could merely reflect the bargaining power of labor and capital and, as

[20] Hart suggests that this may possibly be due to a reorganization of labor agreed to by the unions. To the extent that this leads to increased efficiency and output, then the causation could be said to run from right to left.

we have argued above, can change without there necessarily being any change in the technical conditions of production.

To discuss the decomposition of output growth in any "meaningful" way into TFPG and the contribution of the growth of factor inputs *does* require aggregate production theory, *pace* Hart. This is true whether we measure TFPG in terms of the weighted growth of factor prices or in terms of the growth of output minus the weighted growth of factor inputs. Indeed, the two methods will, as we have seen, give identical results if r or J is calculated residually (as Hart does). Any differences will be purely statistical due to different data sources, etc. Indeed, Hart's empirical analysis comes largely to this conclusion and ironically his method would be the same procedure as that adopted by a neoclassical growth accountant working from the dual.

Hart (1996) also confusingly argues that when one uses gross output and double deflation, the identity no longer holds. "Instead we write $Y = f(L,M,K)$ and we use production theory" (Hart 1996, p.229), where Y is gross output and M is materials It is difficult to see any justification for this as one can get an implicit deflator for wages and profits. Hart concedes that analysis using the production function has also to use deflated values "so the problems created by relative changes in output and input prices are still present" (Hart 1996, p…). So, according to this argument, an index number problem necessitates a particular theory, which is subject to the same problem. Index number problems do present problems for quantitative analysis but they are only of second-order importance in this context. For example, in the case of gross output, there are no separate price deflators for the wages and the rate of profit, so an implicit price deflator can be constructed for these (indeed, should be constructed) to preserve the identity in real terms.

Finally, as noted in the introduction, it should be noted that estimates of TFPG have recently been calculated using physical output at the seven-digit SIC product classification (e.g., Foster, *et al.* 2005). An example of this type of output is ready-mixed concrete or boxes. However, this approach does not avoid our strictures, as inputs are still measured in value terms and the weights are the cost shares. Thus, total factor productivity growth in value terms is given by

$$TFPG_{it} = (\hat{Q}_{it} + \hat{p}_{it}) - a_{L_{it}} \hat{L}_{it} - a_{J_{it}} \hat{J}_{it} - a_{M_{it}} \hat{M}_{it} - a_{E_{it}} \hat{E}_{it} \quad (16)$$

where $\hat{Q}$ is the growth of firm *i*'s physical output, $\hat{p}_{it}$ is the rate of change of firm *i*'s *relative* price and $\hat{M}$ and $\hat{E}$ are the growth rates of materials and energy. The a's are the relevant factor shares. The growth of "physical" total factor productivity is given by $TFPG_{Q_{it}} = TFPG_{it} - \hat{p}_{it}$. While this approach has the advantage of removing from the estimate of total factor productivity growth fluctuations in a firm's relative price due to, for example, "idiosyncratic" demand shocks, it can be seen that the contribution to output growth of the inputs is still calculated using value data.

6. CONCLUSIONS

This paper has taken a radical look at growth accounting exercises. The starting point is the reminder of the damning results for the notion of aggregate production function of the Cambridge-Cambridge capital debates and the aggregation literature, which came to the same conclusion that this concept has no theoretical foundations. Growth accounting exercises, however, take it for granted that an aggregate production function exists. This paper has used simulations to study what growth accounting is actually measuring. In particular, it has asked if there is any relationship between the true Solow residual calculated from micro production functions and that calculated from an aggregate production function. We have used a series of simple simulations exercises to address the question.

The main conclusion is that the true rate of technological progress, calculated with micro data differs from the aggregate, calculated with value data.

How are then we to interpret the Solow residual, in either its primal or dual form? For it to be a seen as a uniquely determined measure of technical progress requires the assumption that the inputs and outputs of the economy (or individual industries) can be represented by an aggregate production function. This seems implausible, given the theoretical objections that have been leveled against it. As Hahn

(1972, cited by Blaug, 1974, p.19) put it: "It has often been the case that a neo-classical theory has been attempted in terms of aggregate production functions and aggregates like capital. Except under absurdly unrealistic assumptions such an aggregate theory cannot be shown to follow from the proper theory and in general is therefore open to severe logical objection. … On purely theoretical grounds there is nothing to be said in its favor. The view that that nonetheless it "may work in practice" sounds a little bogus and in any case the onus of proof is on those who maintain this." Recourse to empirical estimation does not shed any light on the issue, and, indeed, it is always possible to get a perfect fit to the data in a form that resembles an aggregate production function. The simulation exercises have shown how the use of value data will give a misleading impression of the sources of growth. For example, even though the micro-production functions each exhibited increasing returns to scale with labor and capital's output elasticities taking a value of 0.3 and 0.9, respectively, empirical estimation with value data, however, gives the traditional values of 0.75 and 0.25. Moreover, the estimates of TFPG are very different from the true values at the industry level. Barro (1999) has suggested how the growth accounting approach may be adjusted to incorporate the arguments of the endogenous growth models. However, this requires the existence of a well-behaved aggregate production function. The problem is that physical data for industrial processes are not readily available and where it is, it is at a very low level of aggregation. Moreover, it is very unlikely that the simple (if not simplistic) Cobb-Douglas, or even the translog, is likely to adequately model the production processes of a modern manufacturing plant.

Solow once remarked that: "I have never thought of the macroeconomic production function as a rigorous justifiable concept. In my mind, it is either an illuminating parable, or else a mere device for handling data, to be used so long as it gives good empirical results, and to be abandoned as soon as it doesn't, or as soon as something better comes along" (Solow 1966, pp.1259-1260). Perhaps such time has arrived. On the debate about the source of growth in East Asia, Felipe (1999) already warned against what he termed the *Solow residualization* of the East Asian countries, and advocated the application of a completely different methodology to understand the phenomenal growth of the NIEs (e.g., use of firm-level data). And Felipe and Fisher (2003, p.257) concluded their survey on the aggregation literature as follows: “Macroeconomists should pause before continuing to do applied work with no sound foundation and dedicate some time to studying other approaches to value, distribution, employment, growth, technical progress, etc., in order to understand which questions can legitimately be posed to the empirical aggregate data.” The neoclassical growth accounting exercise merely serves to reinforce these conclusions.

References:

Barro, Robert. 1999. “Notes on Growth Accounting.” Journal of Economic Growth, Vol.64, No.2 (June): 119-137.

Bernanke, Ben S. (1987): “Comment”, *NBER Macroeconomics Annual 1987*, pp.202-205, The MIT Press, Cambridge, MA.

Blaug, M. (1974), *The Cambridge Revolution. Success or Failure? A Critical Analysis of Cambridge Theories of Value and Distribution*. Eastbourne: Institute of Economic Affairs.

Cobb, C.W. and Douglas, Paul, H. 1928. “A Theory of Production.” American Economic Review, Vol.18, Supplement (March), Papers and Proceedings of the Fortieth Annual Meeting of the American Economic Association, (December): pp. 139-165.

Cohen, A. and G.C. Harcourt. 2003. “Whatever Happened to the Capital Controversies?” Journal of Economic Perspectives (Winter): 199-214.

Douglas, Paul. 1976. “The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some Empirical Values.” Journal of Political Economy, (October): pp. 903-915.

Felipe, Jesus. 1999. “Total Factor Productivity Growth in East Asia: A Critical Survey.” The Journal of Development Studies, Vol.34, No.5 (April): 1-41.

Felipe, Jesus and Adams., F. Gerard. 2005. “A THEORY OF PRODUCTION. The Estimation of the Cobb-Douglas Production Function: A Retrospective View.” Eastern Economic Journal.

Felipe, Jesus, and Fisher, Franklin M. 2003. “Aggregation in Production Functions: What Applied Economists Should Know.” Metroeconomica, Vol.54, No.2-3 (May-September): 208-262.

Felipe, Jesus, and Carsten Holz. 2001. “Why Do Aggregate Production Functions Work? Fisher’s Simulations, Shaikh’s Identity, and Some New Results.” International Review of Applied Economics, Vol.15, No.3 (July): 261-285.

Felipe, Jesus, and J.S.L. McCombie. 2005a. “On the Rental Price of Capital and the Profit Rate. The Perils and Pitfalls of Total Factor Productivity Growth.” Manuscript.

Felipe, Jesus, and J.S.L. McCombie. 2005b. “Is a Theory of Total Factor Productivity really Needed?” Manuscript.

Felipe, Jesus, and J.S. L. McCombie. 2005c. "How Sound are the Foundations of the Aggregate Production Function?" Eastern Economic Journal.

Felipe, Jesus, and J.S. L. McCombie. 2003. "Some Methodological Problems with the Neoclassical Analysis of the East Asian Miracle." Cambridge Journal of Economics, Vol.54, No.5 (August): 695-721.

Felipe, Jesus, and J.S.L. McCombie. 2001. "Biased Technical Change, Growth Accounting, and the Conundrum of the East Asian Miracle." Journal of Comparative Economics, Vol.29, No.3 (September): 542-565.

Fernald, John, & Neiman, B. (2003) Measuring Productivity Growth in Asia: Do Market Imperfections Matter?" Federal Reserve Bank of Chicago, WP 2003-15.

Fisher, F.M. 1971a. "Reply", Econometrica, Vol.39, No.2 (March): 405.

Fisher, F.M.1971b. "Aggregate Production Functions and the Explanation of Wages: A Simulation Experiment", The Review of Economics and Statistics, Vol.53, No.4 (November), pp.305-325.

Fisher, F.M., Robert Solow, and J.M. Kearl. 1977. "Aggregate Production Functions: Some CES Experiments." Review of Economic Studies, Vol.44, No.2: 305-320.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2005. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" NBER Working Paper 11555.

Friedman, Milton. 1953. "The Methodology of Positive Economics." In M. Friedman (ed.) *Essays in Positive Economics*. Chicago: Chicago University Press.

Hart, Peter E. 1996. "Accounting for the economic growth of firms in the UK manufacturing since 1973." Cambridge Journal of Economics, 20: 225-242.

Hsieh, Chang-Tai. 1999. "Productivity Growth and Factor Prices in East Asia." American Economic Review. Papers and Proceedings, May, Vol.89, No.2: 133-138..

________. 2002. "What Explains the Industrial Revolution in East Asia? Evidence from Factor Markets." American Economic Review, 92, pp.502-526.

Hulten, C.R.. 2000. "Total Factor Productivity: A Short Biography." Working Paper No.7471, Cambridge, MA: National Bureau of Economic Research.

Jorgenson, D.W. 1963. "Capital Theory and Investment Behavior." American Economic Review Papers and Proceedings (May): 247-259.

Jorgenson, D. W., and Z. Griliches. 1967. "The Explanation of Productivity Change." Review of Economic Studies 34, No.3, (July): 249-83.

Kim, J-I and Lau, L. 1994. "The Sources of Economic Growth of the East Asian Newly Industrialized Countries." Journal of the Japanese and International Economies, Vol. 8: 235-271.

McCombie, JSL. 1998a. "Are There Laws of Production?': an assessment of the early criticisms of the Cobb-Douglas production function. Review of Political Economy, Vol.10, No.2 (April): pp. 141-173.

McCombie, JSL. 1998b. "Rhetoric, Paradigms and the Relevance of the aggregate Production Function." In P. Arestis (ed.), *Method, Theory and Policy in Keynes. Essays in Honour of Pul Davidson, Volume III*, Cheltenham, Edward Elgar.

Mohr, Michael F. 1986. "The Theory and Measurement of the Rental Price of Capital in Industry-Specific Productivity Analysis: A Vintage Rental Price of Capital Model." In Ali Dogramaci (ed.), *Measurement Issues and behavior of Productivity Variables*, pp.99-159. Boston/Dordretch/Lancaster: Kluwer Nijhoff Publishing.

Nadiri,I. 1970. "Some Approaches to the Theory and Measurement of Total Factor Productivity: A Survey." Journal of Economic Literature, 8: 1137-1377.

Nelson, Richard, and Howard Pack. 1999. Nelson, Richard R., and Pack, Howard, "The Asian Growth Miracle and Modern Growth Theory." Economic Journal, 109 (July), 457: 416-436.

Prescott, Edward. 1998. "Needed: A Theory of Total Factor Productivity." International Economic Review, Vol.39, No.3 (August): 525-551.

Shaikh, Anwar. 1980. **"**Laws of Production and Laws of Algebra: Humbug II." In E. J. Nell (ed.) *Growth, Profits and Property, Essays in the Revival of Political Economy*, pp.80-95. Cambridge: Cambridge University Press.

Solow, Robert. 1966. "Review of *Capital and Growth*", American Economic Review, Vol.56, No.5 (December): 1257-1260.

Sylos Labini, Paolo. 1995. "Why the Interpretation of the Cobb-Douglas Production Function Must be Radically Changed", Structural Change and Economic Dynamics, Vol.6: 485-504.

Young, Alwyn. 1992. "A Tale of Two Cities: Factor Accumulation and Technical Change in Hong-Kong and Singapore." In Olivier J. Blanchard and Stanley Fischer, Eds., *National Bureau of Economic Research Macroeconomics Annual*: 13-54, Cambridge and London: MIT Press.

Young, Alwyn. 1995. "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience." Quarterly Journal of Economics, 110, 3 (August): 641-680.

Young, Alwyn. 1998. "Alternative Estimates of Productivity Growth in the NICs: A Comment on the Findings of Chang-Tai Hsieh." National Bureau of Economic Research (Cambridge, MA), Working Paper No. 6657.