

Department of Mathematics and Statistics

Topics for the IPh.D. Comprehensive Examination (Mathematics as Coordinating Discipline only)

Approved by the Graduate Committee on 10/27/2005

I. Algebra

Groups: Subgroups, cosets, homomorphisms, quotient groups, Lagrange's Theorem. Normal subgroups, characteristic subgroups, automorphisms. Simple groups.

Actions of groups on sets. Orbits, transitivity, stabilizers, conjugacy classes. The Sylow Theorems.

Rings: Basics about commutative rings with identity, ideals, prime and maximal ideals, Principal Ideal Domains (PID), Unique Factorization Domains and Euclidean rings.

Modules: Vector spaces; basis and change of basis. Basics about modules, linear maps and matrices, submodules, direct sums and free modules.

Structure theory of finitely generated modules over a PID. In particular, classification of finitely generated abelian groups.

Field extensions: simple/algebraic/separable extensions. Multiplicativity of degree. The theorem of the primitive element. Algebraic closures.

Galois theory: splitting fields, normal extensions, Galois extensions. The fundamental correspondence. Cyclotomic extensions of rationals. Finite fields and Galois groups.

II. Analysis

Real variables: Outer measure. Countable subadditivity. Measurable sets, measurable functions and Egoroff's theorem. Vitali Cover Theorem. Lebesgue Density Theorem. Lebesgue's integration and convergence theorems. Differentiation and integration. Monotonicity, convexity, bounded variation and absolute continuity. Radon-Nikodym theorem. Lebesgue's decomposition. Product measures. Fubini's Theorem and Tonelli's Theorem. Banach spaces. L^p spaces, Hölder's inequality and Riesz Representation Theorem. Separable metric spaces and Ascoli-Arzelà theorem.

Complex variables: Power series and analytic functions. Maximum modulus principle and some applications. Cauchy's theorem and Cauchy's Formula. Isolated singularities. Laurent series. Conformal mappings. Linear fractional transformation.

III. Applied Mathematics

Inner product space, projectors, orthogonal projectors, least-squares problem, QR-factorization, Gram-Schmidt method, Householder method, vector and matrix norms, symmetric eigenvalue problems, positive definite and semi-definite matrices, singular value decomposition, perturbation theory for least-squares solution, pseudo inverses, data compression

Best approximations in a normed linear space, approximations from spline function space, linear programming, solutions of nonlinear system, Broyden's method, nonlinear least-squares problem, Gauss-Newton method, boundary value problem, finite element method, Galerkin method, variational method, finite difference method, shooting method, multiple shooting method

IV. Probability

Probability measures, sigma-fields; construction and extension of probability measure on a field and on a sigma-field; the uniqueness and the pi-lambda theorem and Monotone Class theorem.

General measure and its uniqueness; outer measure, and the extension theorem; measures in Euclidean space; measurable functions and mappings; limits and measurability; transformation of measures.

Integral, definition, nonnegative functions, and uniqueness; properties of integral such as equalities and inequalities, integration to the limit, integration over sets, change of variable and uniform integrability; product spaces, product measure, Fubini theorem and integration by parts.

Random variables and vectors, sequences of random variables; independence; four types of convergence of random variables: a.s, in probability, L^p , in distribution and their relationship; Expected values and their properties, moment generating functions; strong and weak law of large numbers, zero-one law, maximal inequalities, convergence of random series; Poisson process and its characterizations.

Convergence in distribution or weak convergence and its properties such as Helly's theorem; characteristic functions, inversion and the uniqueness theorem, the continuity theorem; central limit theorem for i.i.d case and Lindeberg and Lyapounov theorems; infinitely divisible distributions; central limit theorems for R^K .

Radon-Nikodym Theorem, conditional probability, conditional expectation and its properties; martingales, stopping times, and their properties such as Doob's optional sampling theorem, Doob's maximal inequality, up-crossing inequality, (sub-)martingale convergence theorems.

V. Statistics

Random variables; discrete distributions and their properties; continuous distributions and their properties; joint distributions; marginal distributions; conditional distributions; independent random variables and their properties; bivariate normal distributions and their properties; mathematical expectations and their properties; correlation; covariance; conditional expectation; moment generating functions and their properties; joint moment generating functions and their properties; transformation of random variables; order statistics (in particular the extreme order statistics) and their distributions.

The central limit theorem; stochastic convergence and its properties; sampling distributions; point estimation; methods of estimation (MME and MLE); large sample properties of point estimator; Unbiasedness; UMVUE; Cramer-Rao Lower Bound (CRLB); MSE; sufficient statistics and their properties; completeness and exponential class; Rao-Blackwell theorem; Lehmann-Scheffe theorem; tests of hypotheses; size of a test; power of a test; Lehmann-Pearson Lemma; MP test; UMP test; MLR property of a pdf; GLR test.